**NUMERICAL ANALYSIS**

It is hard to describe our real-life problems with sophisticated mathematical formulas. Most of the cases complicated problems arise in a differential formation. Then finding the solution for this kind of equation is obvious. But most of the time it is hard to find out the exact solution of such kind of problems and sometimes there is no exact solution for the problems. In this scenario we need to estimate our best guess for the problem. There is various methodology to find out the most exact numerical value for the problems. Though we are not getting the exact value but we are very much close to the exact solution and sometimes the solution is nicely fitted. In 1614 after the creation of logarithms by John Napier, it has been an important aspect for numerical analysis.

**HISTORY OF NUMERICAL ANALYSIS:[4]**

A simple method for finding the root of a simple equation is found in Egyptian Rhind papyrus in 1650 BC. Next Eudoxus of Cnidus and Archimedes estimated the method for calculating areas volumes and length of any geometric figures. When Isaac Newton and Gottfried Leibniz developed the calculus, it has been a great tool to solve numerical problems. Calculus led to accurate mathematical models for physical reality, first in the physical sciences and eventually in the other sciences, engineering, medicine, and business. These mathematical models are usually too complicated to be solved explicitly, and the effort to obtain approximate, but highly useful, solutions gave a major impetus to numerical analysis. Newton created a number of numerical methods for solving a variety of problems, and his name is still attached to many generalizations of his original ideas. Particularly finding roots for general functions and finding a polynomial equation which fits best with the data that is polynomial interpolation. After Newton’s mathematician Swiss Leonhard Euler (1707–1783), the French Joseph-Louis Lagrange (1736–1813), and the German Carl Friedrich Gauss (1777–1855) contribute to the numerical analysis in 18th and 19th century.

After the creation of computer, numerical analysis become sophisticated. Critical and more complex analysis could be possible to do with the help of computer. Now it is impossible to solve numerical problems without computer.

**METHODS OF NUMERICAL ANALYSIS:[2]**

Depending on the problems we have to use various types of method for numerical analysis. Here we will consider the one variable equations. The most basic methods are

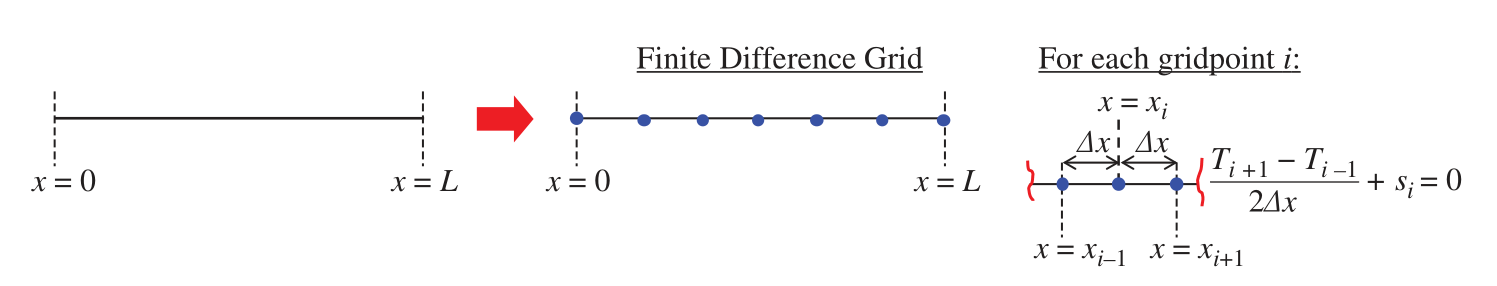
**FINITE ELEMENT METHOD [3]**

Before understanding finite element method let’s understand finite different method. Consider the differential equation

Where T(x) is unknown function which we need to find out and s(x) is a given function.

To find out the solution let us consider the central difference formula. Let divide the domain into finite interval with some grid points with equal space . For each location grid point , we use the two nonboring points , to write the following approximation of the derivative at ,

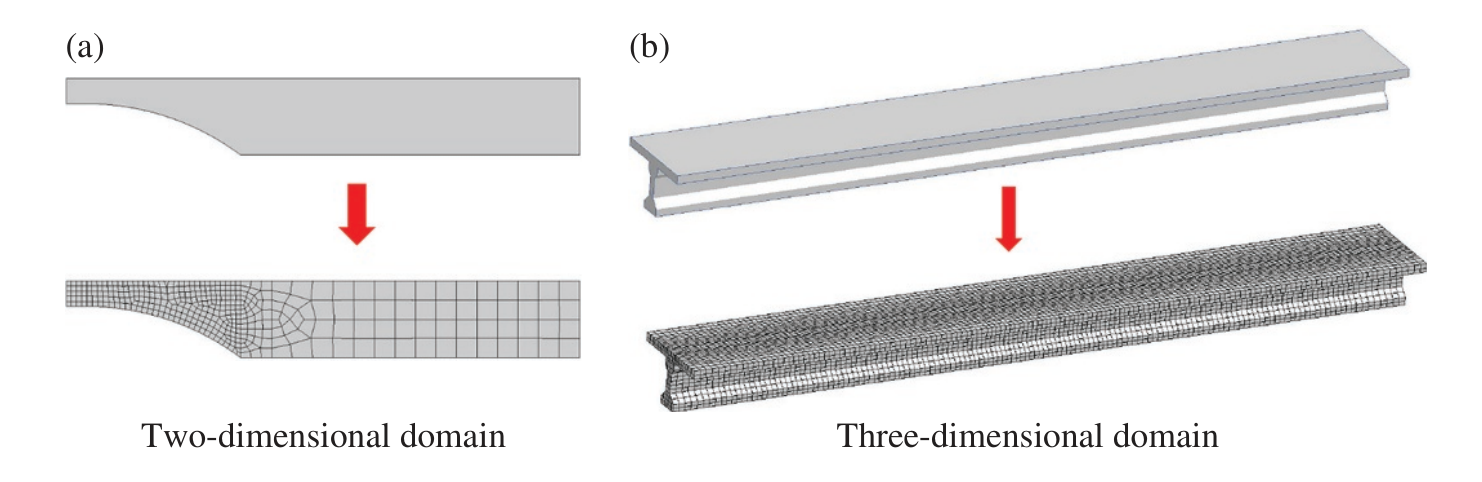
Now putting this value in the equation, we get



**figure**

Where . For each grid point we will get a system of equation which will allow us to find out the unknown function T(x).

In Finite Element method first, we need to discretize or subdivide the physical geometric domain into smaller subdomains which are called elements. Now the function governs each element are called field functions. We have to setup the field function, usually these functions are polynomial which contain some parameter which we need to obtain.



**HISTORY OF FINITE ELEMENT METHOD [3]**

It is not clear who introduced finite element method first. But it is accepted that this method is developed from aerospace engineering research in the 1950s. Professors Jon Turner (in the United States) and John Argyris (in Europe) are the two initial contributors of finite element method. In 1953, Turner came up with the idea of pursuing improved models of aircraft wings through representation of the skin with triangular elements, having constant stress. After some years Turner el al., Clough realized the importance of direct stiffness method. Next time they named it finite element method. In 1960 he published a paper mentioning the name. Major early contributions on finite element analysis also came from Professor Olgierd Zienkiewicz at the University of Swansea, Wales, and his collaborators. After the mathematical rigorous verification of the validity of the FEM, the method became vastly more popular, primarily thanks to research conducted at UC Berkeley by Taylor and Wilson.

**BASIC ALGORITHM OF FINDING THE FINITE ELEMENT SOLUTION: [3]**

\*First, we need to define the Governing differential equation for the physical problem with the boundary condition which is known as **strong form**.

Where E(x) → Young’s modulus, A(x) → cross-sectional area of a metal road where force is applied to contract or stress the metal road.

\*Then we have to consider a more convenient equation as a solution which is closer to the main solution which is called the weak form.

**\***Next the finite element approximation is used.

Here [N] is a matrix with known function of x which are chosen and {U} are unknown parameters which have to determine.

\*If we plug the approximation in the weak form, we eventually obtain an algebraic system of linear equations which we can solve for {U}.

**VARIOUS FINITE ELEMENT METHODS:[5]**

Consider the boundary value problem

Now consider the approximate trial solution as

Which satisfy the boundary condition and in a simple form. Here is a parameter. It is clear that  . Now we have to find out the value of for which we can get the best result. Now the main equation can be wrriten as

If we put the approximate value in this equation then we will not get zero. So, putting the approximate solution we can write

This function is called the residual function. It is clear that if the approximate solution coincides with the exact solution, then for all x in the domain.

A number of techniques based on the use of residual functions are available for determining the unknown parameters in the trial solutions.

One such method is called the collection method which involves forcing the approximate solution to be exact at n points where n is the number of unknown parameters. This implies that

Which is a set of n equations to be solved for the parameters.

**Galerkin’s Weighted Residual Method:**

Galerkin’s method involves the concept of weighted residuals. In this method we determine the n unknown parameters by selecting n weighting functions and requiring that each of the n integrals

In Galerkin’s method we use as weighting functions those terms in the trial solution which are multiplied by unknown parameters. Considering the trial function, we can take

So, the residual function can be written as

Now the Galerkin procedure requires to be such that

So, we get

After doing the integration we get

Then our approximation solution is

Now the exact solution to this boundary value problem is known and the exact solution is

The graph of both trial and exact solution with the residual function are shown in the graph.

Gaph graph graph graph graph ……………………………..

The formal statement of the Galerkin approximation procedure at least for linear second order one dimensional boundary value problems of the general form:

With given boundary conditions as x = a and x = b

Approximate trial solutions are used which are of the form

Here

* is chosen so that it satisfies the boundary conditions of the problem.
* The functions must each satisfy the corresponding homogeneous form of the boundary conditions. These functions are called coordinate functions.

**GALERKIN METHOD [1]**

In the area of numerical analysis, Galerkin Methods is used to convert a continuous operator problem to a discrete problem**.** Most of the cases the problems like differential equation commonly in a weak formulation. There are some verities in Galerkin method.

**HISTORY OF GALERKIN METHODS:**

**PREVIOUS WORK ON GALERKIN METHOD:**

**VARIOUS TYPEPS OF GALERKIN METHODS:**

**NORMARL GALERKIN METHOD:**

**MODIFIED GALERKIN METHOD:**

**DISCONTINUOUS GALERKIN METHOD:**

**RITZ-GALERKIN METHOD:[1]**

This method is named after Walter Ritz. It is also called Rayleigh-Riz method and the Ritz-Galerkin method. The Ritz-Galerkin method is a direct method for finding an approximated solution for boundary value problems. First a wave function is assumed which satisfies the boundary condition of the problem. The function contains some adjustable parameter which is used to find out the satisfied solution.

**DISCONTINUOUS GALERKIN METHOD:**

**HISTORY OF DG METHOD:**

**WHY DG METHOD:**

**ALGORITHM OF DG METHOD:**

**AN EXAMPLE ON DG METHODS:**

**CENTRAL DIFFERENCE FORMULA**

There are a lot of numerical method to solve differential equation. Now we are going to define central difference method. Consider the following differential equation,

Where are known function, which are continuous in the interval [a, b]. are single and double derivative of y. Let N>0 be an integer and divide the interval [a, b] into (N+1) equal subinterval. The end points of each interval are the mesh points. Let the mesh points , where . The cause of Taking the subinterval N+1 so that a Linear system can be formed with matrix can be formed.

Now at the mesh points the approximate differential equation can be written as

Now by Taylor polynomial expansion we can get,

For some , and

For some .

Now adding (4) and (5) we get

Now using intermediate value Theorem, we can write

For some ,

Now we get the value of from (6)

Now similarly subtracting (4) and (5) we get the value of as,

For some in .

Now putting this value in (1) we get,

Let , then with the truncation error or order results by using this equation together with the boundary conditions to define the system of linear equations

And

for each i = 1, 2, . . ., N.

From (11) we can represent the equation as

For i=1,2, 3, … N, the resulting system of (12)j can be written in the tridiagonal matrix form,

Where,

By solving the matrix, we can get the approximate values in the mesh points.

This is for linear differential equation. Now consider the partial differential equation.

On , with for , where S is the boundary of R. If f and g are continuous on their domains, then there is a unique solution to this equation.

For this two-dimensional problem we have to choose integer N and M greater than zero. Let’s divide the interval [a, b] into N subinterval and [c, d] into M subinterval. Then the mesh points can be written as

Where .

And are the grid lines and the intersection points of the grid lines are the mesh points for this 2d system. So similarly like the linear problem by Tailor series we can write the centered-difference formula for x variable about ,

Here , also for y variable about , after Tailor series expansion we get the centered-difference formula

Where

Putting this values in (13) we get

for each i = 1, 2, . . . , n − 1 and j = 1, 2, . . . , m − 1. The boundary conditions are

Now the finite difference method is

for each i = 1, 2, . . ., n − 1 and j = 1, 2, . . ., m − 1, and

where approximates . This method has local truncation error of order

**AN EXAMPLE ON CENTRAL DIFFERENCE FORMULA:**

**COMPARISN BETWEEN THE TWO METHODS MATH:**

**CONCLUSION:**

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**4.Britanica**

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